Twin Primes

Tracking Mathematical Siblings Through Computational Deep Space

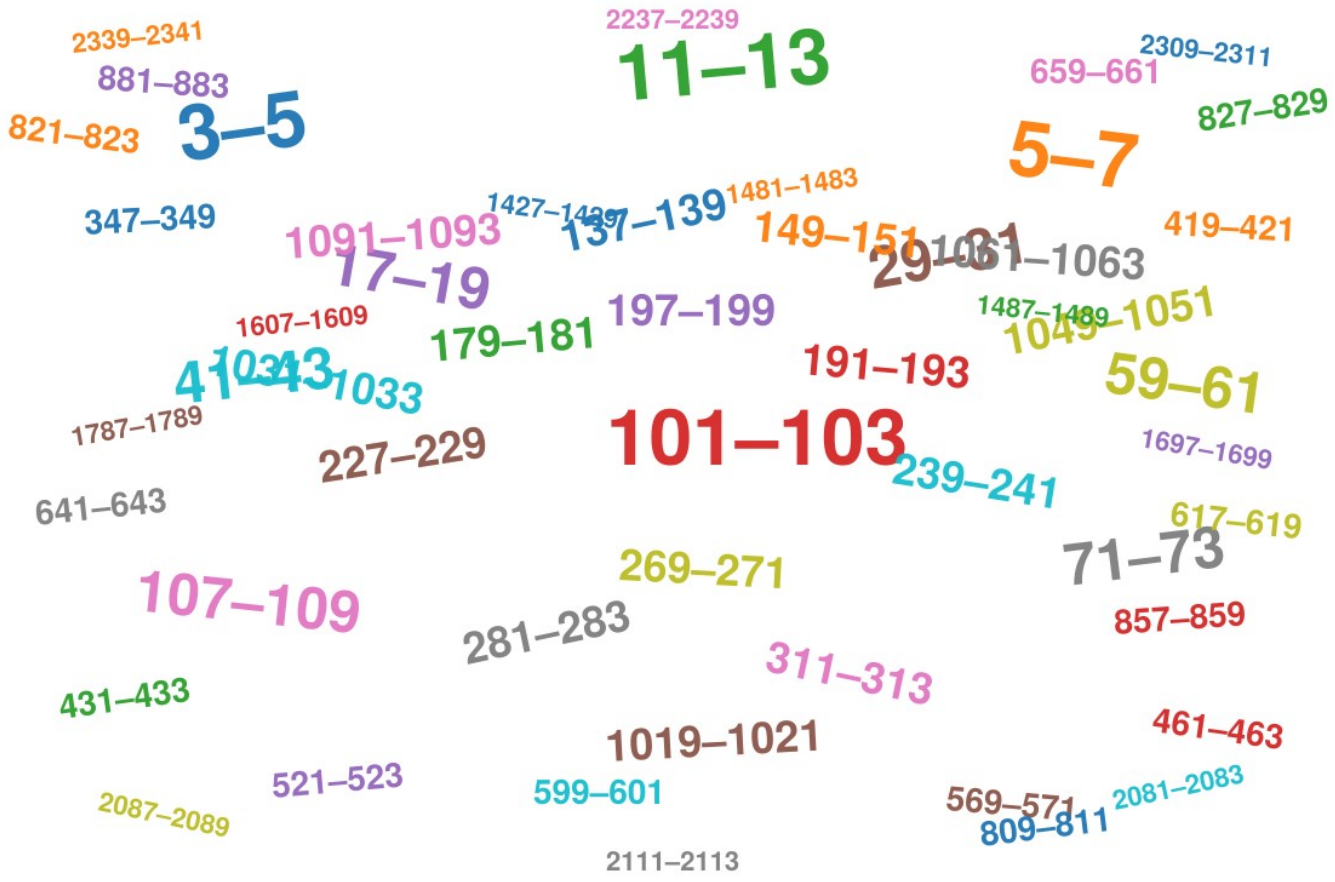
Date 23/10/2025

une
University of New England

What is a Twin Prime?

- A prime is a whole number greater than 1 that can only be divided evenly by 1 and itself
- Twin primes are pairs of primes that differ by 2

How many are there?



Why count them?

$$\pi_2(x) \approx 2 C_2 Li_2(x)$$

Where:

$$C_2 \approx 0.6601618$$

$$Li_2 = \int_2^x \frac{dt}{(\log t)^2}$$

Sieve of Eratosthenes

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
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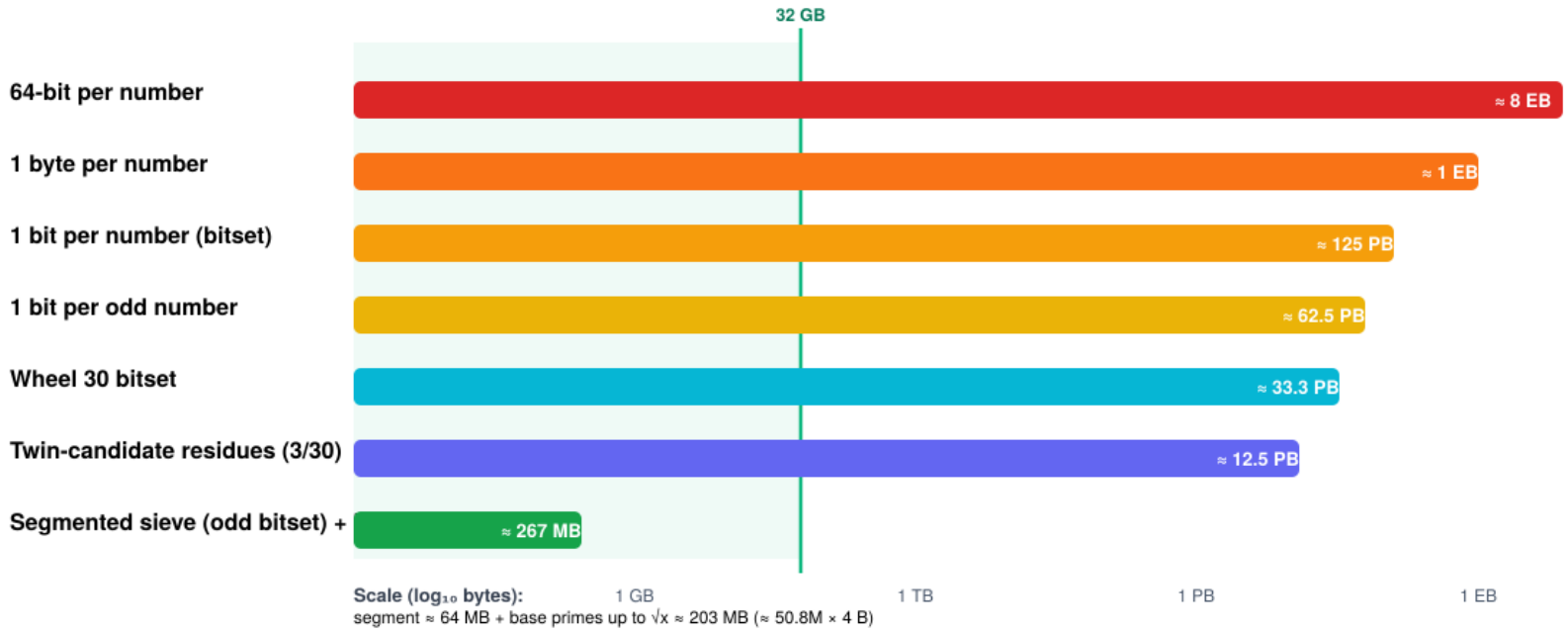
Wheel Factorisation

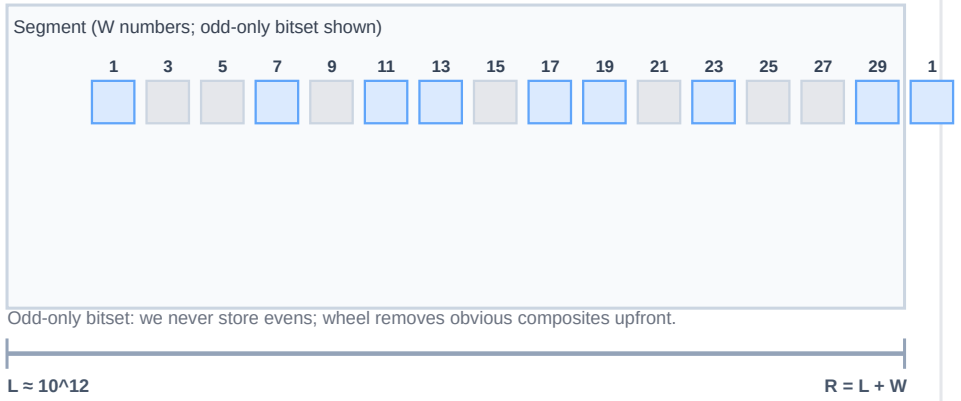
How big is 10^{18}?



Decimal units: 1 PB = 10^15 bytes, 1 EB = 10^18 bytes. x = 10^18.

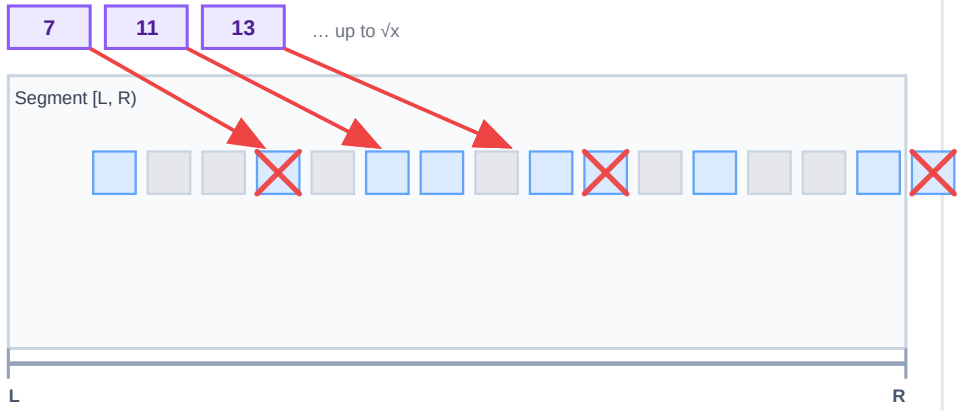
1) Pre-sieve candidates (wheel 30)

Keep only residues {1,7,11,13,17,19,23,29} mod 30; mark twin starts {11,17,29}.



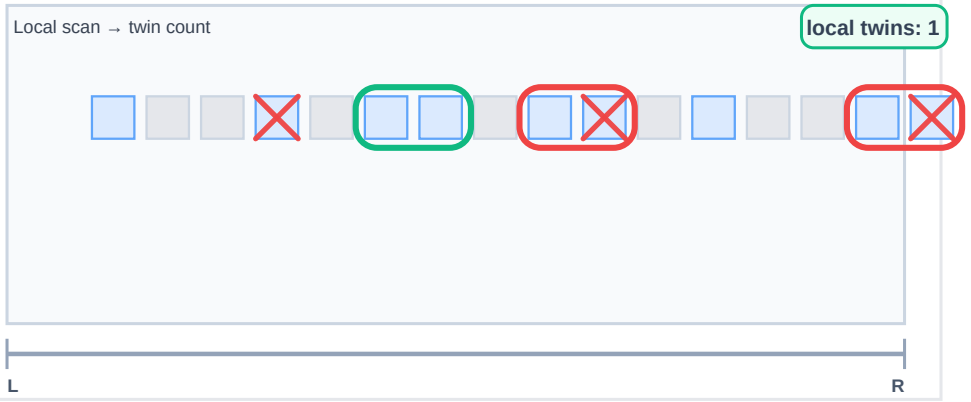
2) Mark composites using base primes ≤ √x

Re-use the same base primes for every segment; cross off their multiples within [L, R).



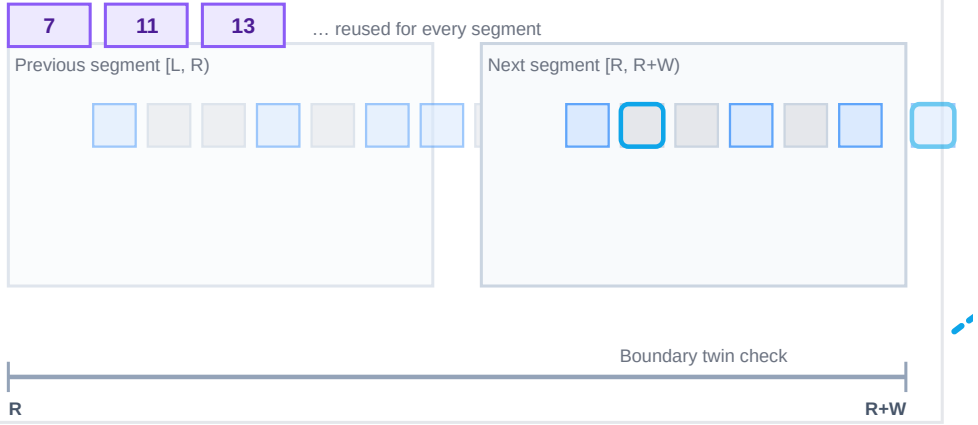
3) Count twin pairs in the segment

Check only p with residues {11,17,29}; twin if both p and p+2 survive.



4) Slide to next segment and check boundary

Reuse base primes; check the pair straddling R (p=R−1, p+2=R+1) once.



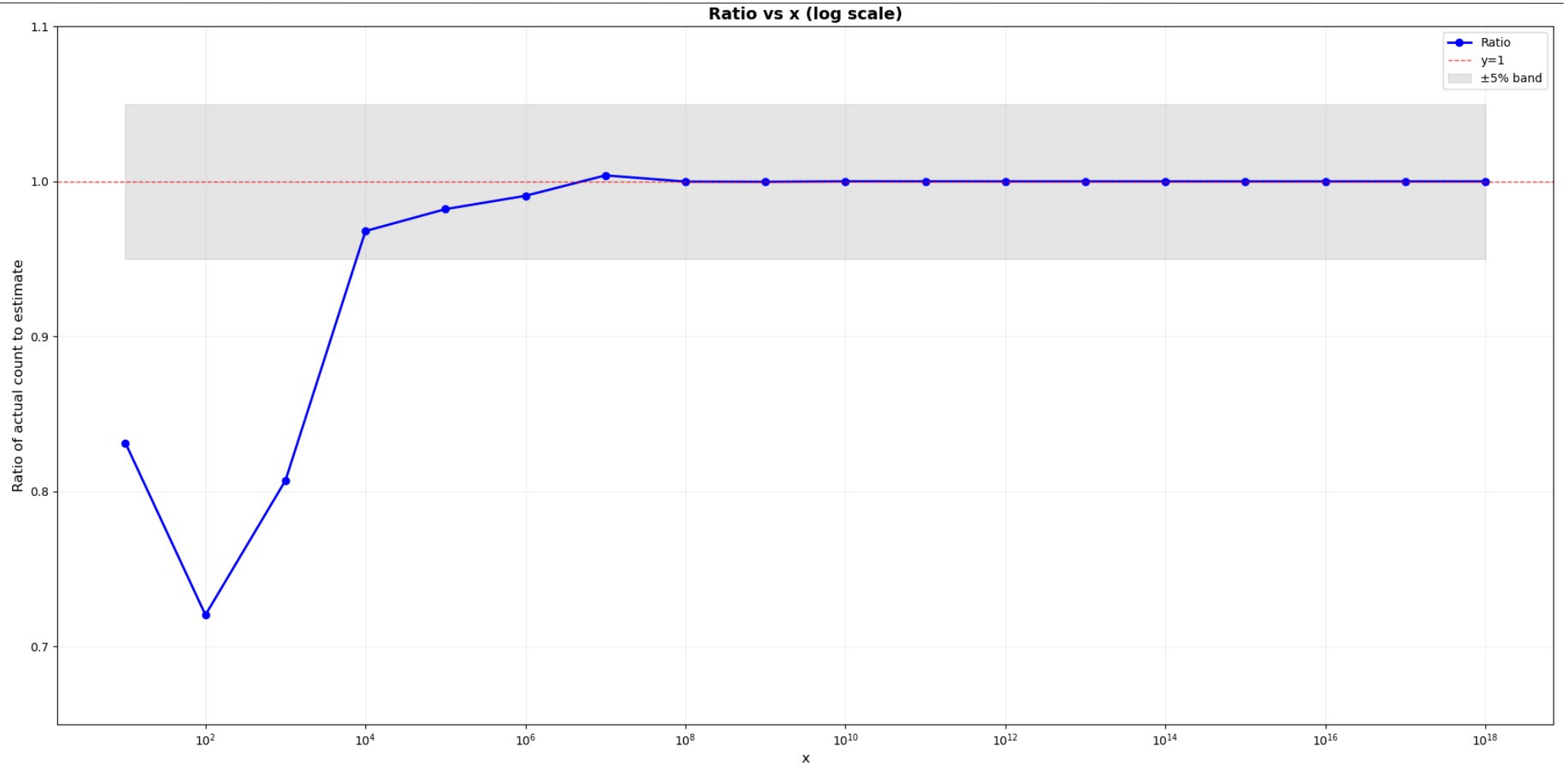
candidate (allowed mod 30) | filtered out (even or multiple of 3 or 5) | twin-start residues {11,17,29} | composite (crossed off) | twin found

Making it fast

- At this scale, details matter!
  - It’s essentially the same algorithm, but careful layout turns weeks into days

- Some techniques:
  - Bit packing
  - Pre-sieving by small primes
  - Be careful with boundaries!
  - Parallelism

What do we see?



Next Steps

- Validation
- Algorithmic improvements
- Different gaps
- Prime constellations